Resit / Numerical Mathematics 1 / July 7th 2020, University of Groningen

A simple calculator is allowed.
You are allowed to use the material of the course (lecture notes, tutorials).
All answers need to be justified using mathematical arguments.
Total time: 3 hour 30 minutes (time includes upload of the PDF with your answers to Nestor) + 30 minutes (if special needs)
Remember: oral "checks" may be run afterwards.

Exercise 2 (9 points)

Consider the function $f(x) = x^3 - 4x^2 + 3$.

(a) $\boxed{1.5}$ Show that there exists $x^* \in [3, 4]$ such that $f(x^*) = 0$.

(b) $\boxed{1}$ Show that x^* is fixed point of the functions

$$g_1(x) = 4 - \frac{3}{x^2}\ ,\ g_2(x) = \frac{1}{4}\left(x^2 + \frac{3}{x}\right).$$

(c) $\boxed{2}$ Show that there exists a vicinity around x^* such that the iterations $g_1(x^{(k)})$ converge to x^* for $x^(0)$ in that vicinity.

(d) $\boxed{1.5}$ What can you say about the convergence to x^* when using g_2?

Consider solving the eigenvalue problem $Ax = \lambda x$, with $A \in \mathbb{R}^{n\times n}$ and symmetric positive definite and eigenvalues all different of each other, using the following iterative procedure

$$x_k = Ax_{k-1}, \quad k \geq 1,\ x_0 \text{ given.}$$

(e) $\boxed{3}$ Show that the sequence x_k converges to the largest eigenvalue of A for any choice of x_0. Hint: remember that the eigenvectors of A form a basis in $\mathbb{R}^n$, hence x_0 can be written as a linear combination of those eigenvectors.